Towards new relativistic hydrodynamcis from AdS/CFT

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with Edward Shuryak

- **QGP is Deconfined**

- **QGP is strongly coupled (sQGP)**

behaves "almost" like a perfect liquid (Navier-Stokes with very small viscosity)

$$\eta \ \sim \ mean \ \ free \ \ path \ \sim 1/\sigma$$

QCD $\longrightarrow \mathcal{N} = 4$ **SYM (CFT)**

Strong coupling (and large N_c**)** $\rightarrow$ **AdS/CFT** $\rightarrow$ **SUGRA on AdS**$_5$

CFT at finite Temperature $\leftrightarrow$ **AdS Black Hole**

Outline of the talk

- Quick tour to Relativistic hydro
- Brief visit into 5th dimension: Black Hole AdS/CFT
- All order hydro: momenta dependent viscosity

Motivation: Experiments (RHIC) probe systems with finite gradients.

Main Goal:

Introduce higher order dissipative terms in the gradient expansion of $T^{\mu\nu}$

Extract momenta dependent viscosities by matching two-point correlation functions of stress energy tensor with correlation functions computed from BH AdS/CFT.

(when applying to QCD we hope for some universality for transport coefficients)

We propose to use this hydro as a “nonlinear model” for real simulations at RHIC

Relativistic Hydrodynamics

from Landau & Lifshitz V6

Energy momentum tensor

$$T^{\mu\nu} = (\epsilon + P)\, u^{\mu}\, u^{\nu} + P\, g^{\mu\nu} + \Pi^{\mu\nu}$$

u - **velocity field of the fluid** $\quad u^2 = -1 \quad$ P - **Pressure**

$\Pi^{\mu\nu}$ - **tensor of dissipations (ideal fluid:** $\Pi^{\mu\nu} = 0$**)**

$u_{\mu}\, \Pi^{\mu\,\nu} = 0$ - **no dissipation in the local rest frame**

Navier Stokes term (expanding in the velocity gradient)

$$\Pi^{\mu\nu} = -\eta\, (\Delta^{\mu\lambda}\, \nabla_{\lambda}\, u^{\nu} + \Delta^{\nu\lambda}\, \nabla_{\lambda}\, u^{\mu} - \frac{2}{3} \Delta^{\mu\nu}\, \nabla_{\lambda}\, u^{\lambda}) - \xi\, \Delta^{\mu\nu}\, \nabla_{\lambda}\, u^{\lambda}$$

$$\Delta^{\mu\nu} = g^{\mu\nu} + u^{\mu}\, u^{\nu}$$

Energy - momentum conservation:

$$\nabla_\mu T^{\mu\nu} = 0 \qquad \longrightarrow \qquad \mathrm{Navier-Stokes\ Eq.}$$

Conformal invariance

$$T^\mu_\mu = 0 \qquad \longrightarrow \qquad \epsilon = 3\,P \quad \mathrm{and} \quad \xi = 0$$

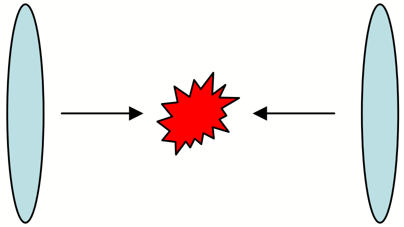
Entropy density and EoS

$$s = \frac{\epsilon + P}{T} = 4\,k_{SB}\,T^3$$

No dissipation no entropy production:

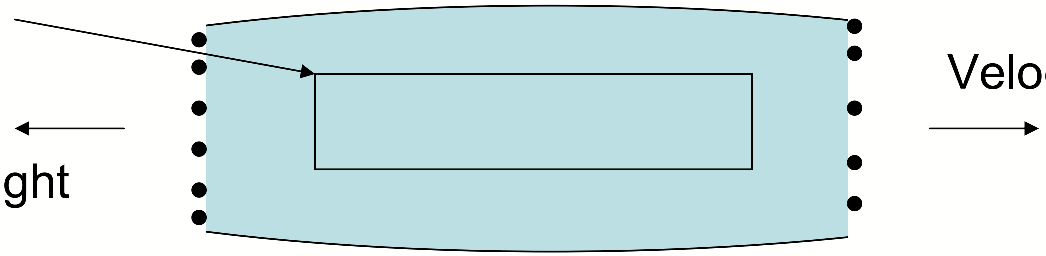
$$\frac{ds}{dt} = 0 \qquad if \qquad \Pi^{\mu\nu} = 0$$

RHIC and Bjorken set up



Relativistically accelerated heavy nuclei

Central Rapidity Region (CRR)



Velocity of light

Velocity of light

After collision

- **one-dimensional expansion**: **Boost Invariant Formulation**

Local rest frame $u = (1, 0, 0, 0)$ $\qquad (x^0,\, x^1,\, x_\perp) \;\longrightarrow\; (\tau,\, y,\, x_\perp)$

τ **- proper time,** $\qquad y$ **- spacetime rapidity**

$$x^0 \;=\; \tau \; ch(y) \qquad\qquad x^1 \;=\; \tau \; sh(y)$$

The metric (1d Hubble expansion)

$$ds^2 \;=\; -\, d^2\tau \;+\; \tau^2 \, d^2 y \;+\; d^2 x_\perp$$

Hydro eq. simplify dramatically:

$$\partial_\tau \, \epsilon(\tau) \;=\; -\,\frac{4\,\epsilon}{3\,\tau} \;+\; \frac{4\,\eta}{3\,\tau^2}$$

Solution for $\eta \;=\; 0$**:** $\qquad$ **Bjorken (1986)**

$$\epsilon \;\sim\; \frac{1}{\tau^{4/3}} \qquad\qquad T \;\sim\; \frac{1}{\tau^{1/3}} \qquad\qquad \partial_\tau \, (s\,\tau) \;=\; 0$$

Sound waves in Relativistic Hydrodynamics

still from Landau & Lifshitz V6

Plane wave perturbation:

$$\delta u \;=\; \delta u_0 \, e^{-i\,\omega\,t \,+\, i\,q\,x} \qquad\qquad \delta P \;=\; \delta P_0 \, e^{-i\,\omega\,t \,+\, i\,q\,x}$$

Linearized Hydro leads to the dispersion relation

$$\omega \;=\; c\,q \;-\; i\,\frac{2\,\eta}{s\,T}\,q^2$$

Sound velocity $c \;=\; 1/\sqrt{3}$ **Sound attenuation** $\sim\; \eta$

Spectral functions in the sound and shear channels ($2\,\pi\,T \;=\; 1$ **and** $\bar{\eta} \;\equiv\; 4\,\pi\,\eta/s$ **)**

$$\chi^L \;=\; \frac{2\,\omega\,c^2\,q^4\,\bar{\eta}}{(\omega^2 \;-\; c^2\,q^2)^2 \;+ 4\,\omega^2\,c^2\,q^4\,\bar{\eta}^2} \qquad\qquad \chi^T \;=\; \frac{\omega\,q^2\,\bar{\eta}/2}{\omega^2 \;+\; q^2\,\bar{\eta}^2/4}$$

Israel-Stewart second order Hydrodynamics

Solves causality problems encoded in Navier-Stokes

Add extra term in the gradient expansion + non-linear terms in (∇u)

$$\Pi^{\mu\nu} = (1 - \tau_R\, u_\lambda \nabla^\lambda)\ \Pi^{\mu\nu}_{NS}$$

Iterate the equation

$$(1 + \tau_R\, u_\lambda \nabla^\lambda)\ \Pi^{\mu\nu} = \Pi^{\mu\nu}_{NS}$$

When thinking about small perturbations $u_\lambda \nabla^\lambda \longrightarrow \nabla_t \longrightarrow -i\,\omega$

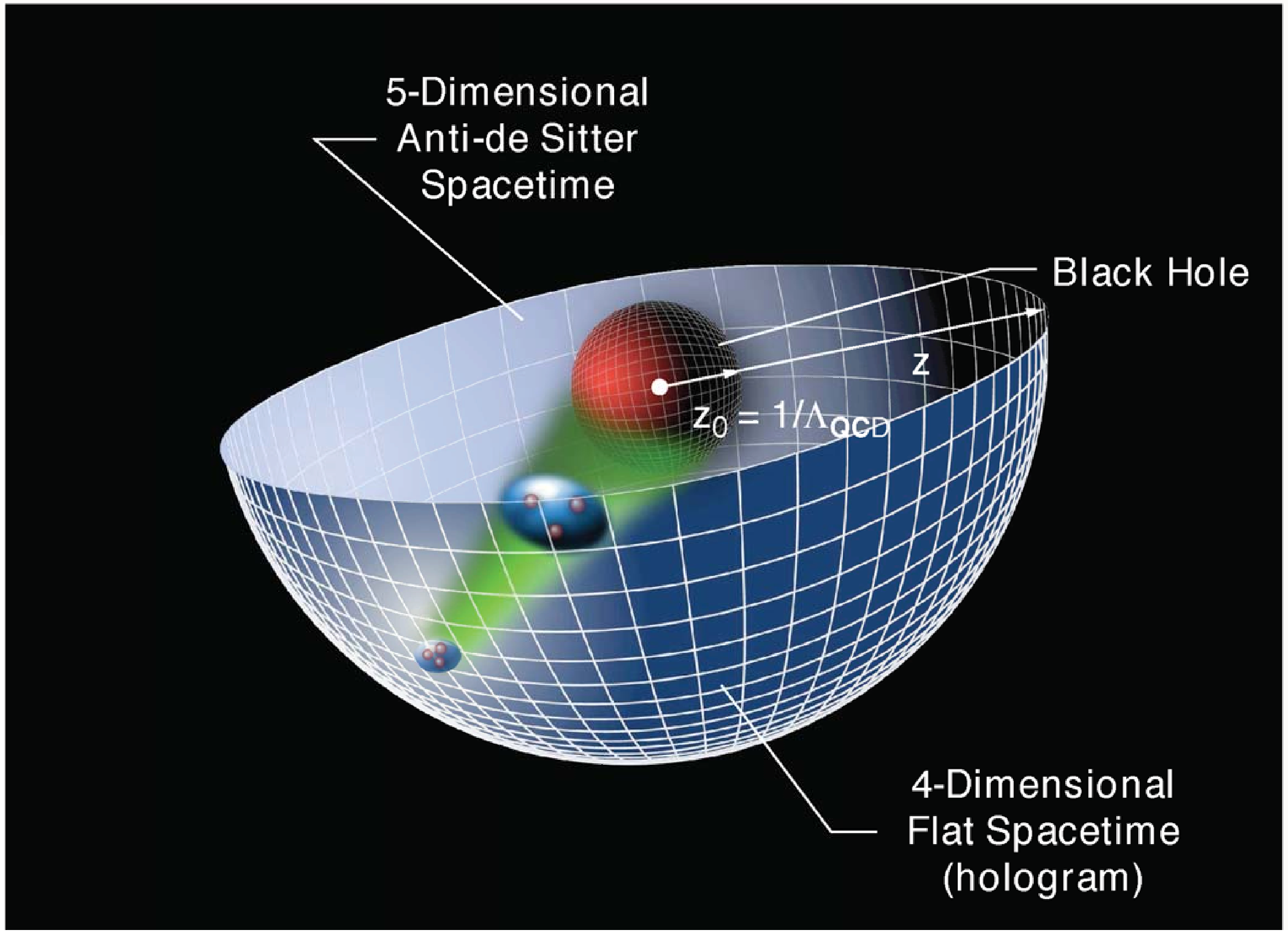
The IS second order hydro is equivalent (in the linear approximation) to

$$\eta \longrightarrow \frac{\eta}{1 - i\,\tau_R\,\omega}$$

Sound dispersion

$$\omega = c\,q\,[1 + \bar\eta\, c^2 q^2\,(2\,\tau_R - \bar\eta)] - i\,c^2\,\bar\eta\, q^2\,[1 + c^2 q^2\,\bar\eta\,\tau_R\,(2\,\bar\eta - \tau_R)]$$

AdS/QCD



Changes in length scale mapped to evolution in the 5th dimension z

picture due to Stan Brodsky

AdS/CFT correspondence: weakly coupled super-gravity in $AdS_5 \times S_5$ is "dual" to strongly coupled $\mathcal{N} = 4$ SYM gauge theory in 4d

AdS_5 Schwarzschild BH metric

$$ds^2 = \frac{\rho^2}{L^2}\left[-\left(1 - \frac{\rho_0^4}{\rho^4}\right) dt^2 + dx^2 + dy^2 + dz^2\right] + \frac{L^2}{\rho^2\,(1 - \rho_0^4/\rho^4)}\, d\rho^2$$

BH Horizon at $\rho = \rho_0$

AdS "boundary" $\rho \rightarrow \infty$ **is Minkowski space (t,x,y,z)**

Gauge theory at the boundary is $\mathcal{N} = 4$ SYM static plasma at finite temperature.

The Hawking temperature is

$$\mathbf{T} = \frac{\rho_0}{\pi\,\mathbf{L^2}}$$

Retarded correlators and Viscosity from AdS BH

Retarded correlators:

$$G_R^{\mu\nu\mu'\nu'}(\omega,\, q) \;=\; -\, i \int_0^\infty dt \int dx\, e^{-\, i\,\omega\, t\, +\, i\, q\, x} \,\langle [T^{\mu\nu}(t, x)\,,\, T^{\mu'\nu'}(0,0)]\rangle$$

AdS/CFT: **energy-momentum tensor $T_{\mu\nu}$ couples at the boundary to metric perturbations (gravitons). Solve linearized GR in 5d with absorptive boundary conditions at the horizon.**

Shear viscosity

$$\eta \;=\; \lim_{\omega \to 0} \frac{1}{2\,\omega} \int dt\, dx\, e^{-\, i\,\omega\, t} \,\langle [T^{xy}(t, x)\,,\, T^{xy}(0,0)]\rangle$$

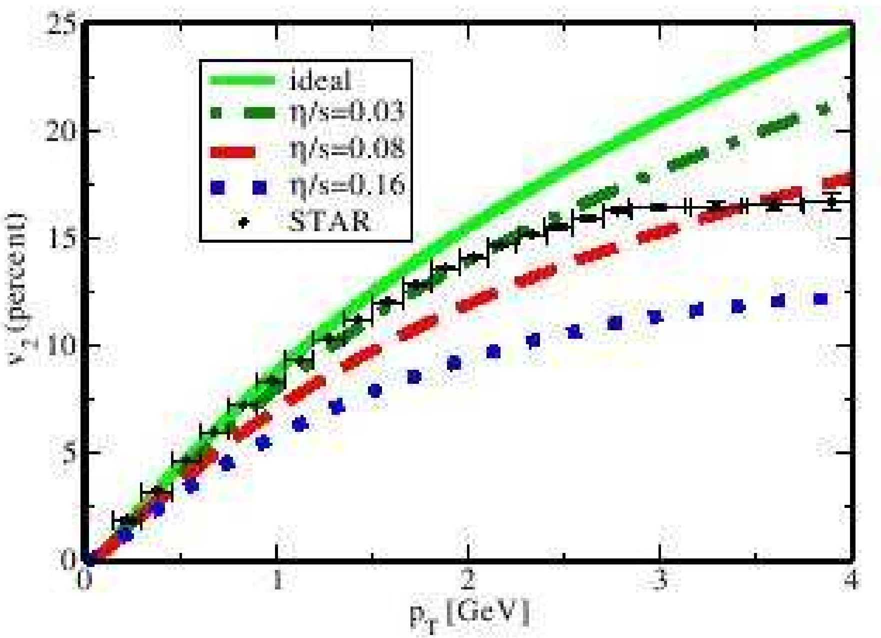
P. Kovtun, D. T. Son and A. O. Starinets, PRL 94, 111601 (2005)

$$\frac{\eta}{s} \;=\; \frac{1}{4\,\pi}$$

Imposing also Dirichlet boundary conditions at the AdS boundary leads to quantization: quasi-normal modes.

Is anything wrong with viscous hydro?

The phenomenologically preffered value for η/s is very small
Viscosity kills the elliptic flow!



P. Romatschke, U. Romatschke, Phys.Rev.Lett.99:172301,2007

Largely supported by H. Song, U. W Heinz, .arXiv:0712.3715
Somewhat disagrees with K. Dusling, D. Teaney arXiv:0710.5932

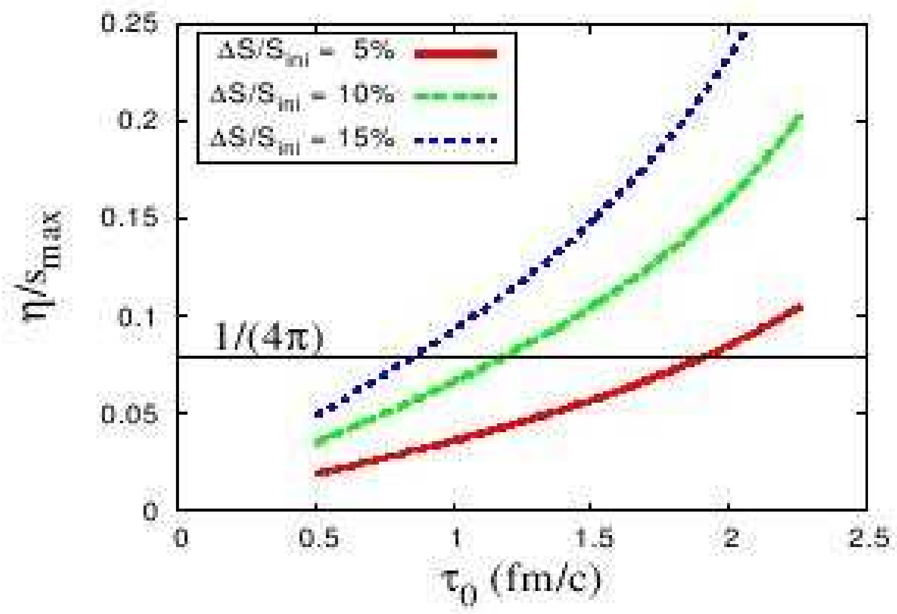
At RHIC Hydro seems to start at very early times $\tau_0 \sim 0.5 - 1\, fm$.

The hydro phase is long (10 fm) $\longrightarrow$ too much entropy is produced by the hydro phase.
That seems to contradict the RHIC data on produced particle multiplicities.

Entropy production in the Bjorken (1d) Hydro

$$\partial_\tau (s\, \tau) = \frac{4\, s}{3}\, \frac{\eta}{s}\, \frac{1}{T\, \tau}$$

A. Dumitru , E. Molnar , Y. Nara, Phys.Rev.C76:024910,2007.



E. Shuryak and M.L., Phys.Rev.C76:021901,2007

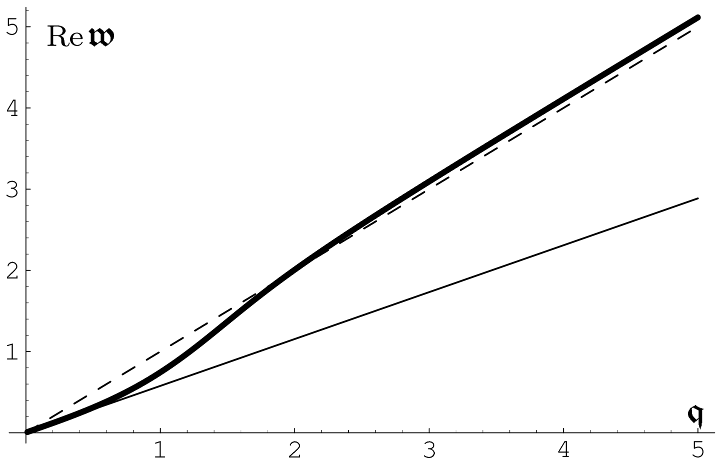
Introduce higher viscosity terms in the gradient expansion of $T^{\mu\nu}$:

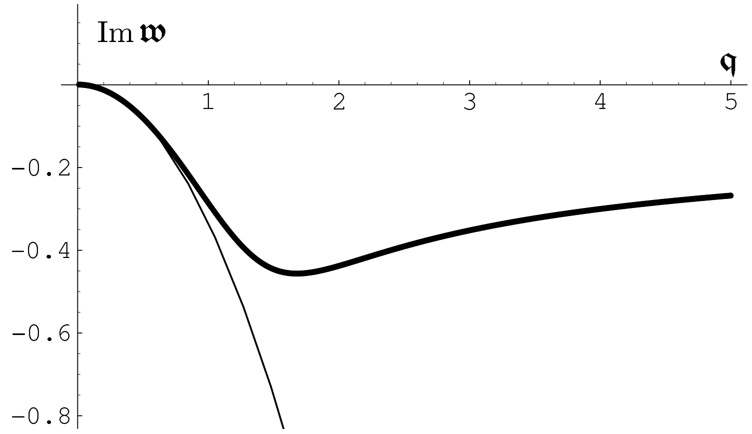
$$\frac{\partial_\tau (s\tau)}{s\, (\tau\, T)} = \frac{\bar{\eta}}{\pi}\left[c^2\, \frac{1}{(\tau\, T)^2} + \sum_{n=2}^{\infty} \frac{\alpha_n}{(T\tau)^{2n}}\right]$$

Sound and Holography

P. Kovtun and A. Starinets, Phys.Rev.D72:086009,2005

Quasi-normal mode analysis in the AdS BH background - the sound channel





$$\Re e[\omega] = c\, q + \sum_{n=1}^{\infty} r_n\, q^{2\,n+1} \qquad\qquad \Im m[\omega] = -\,\bar{\eta}\left[c^2\, q^2 + \sum_{n=2}^{\infty} \beta_n\, q^{2\,n}\right]$$

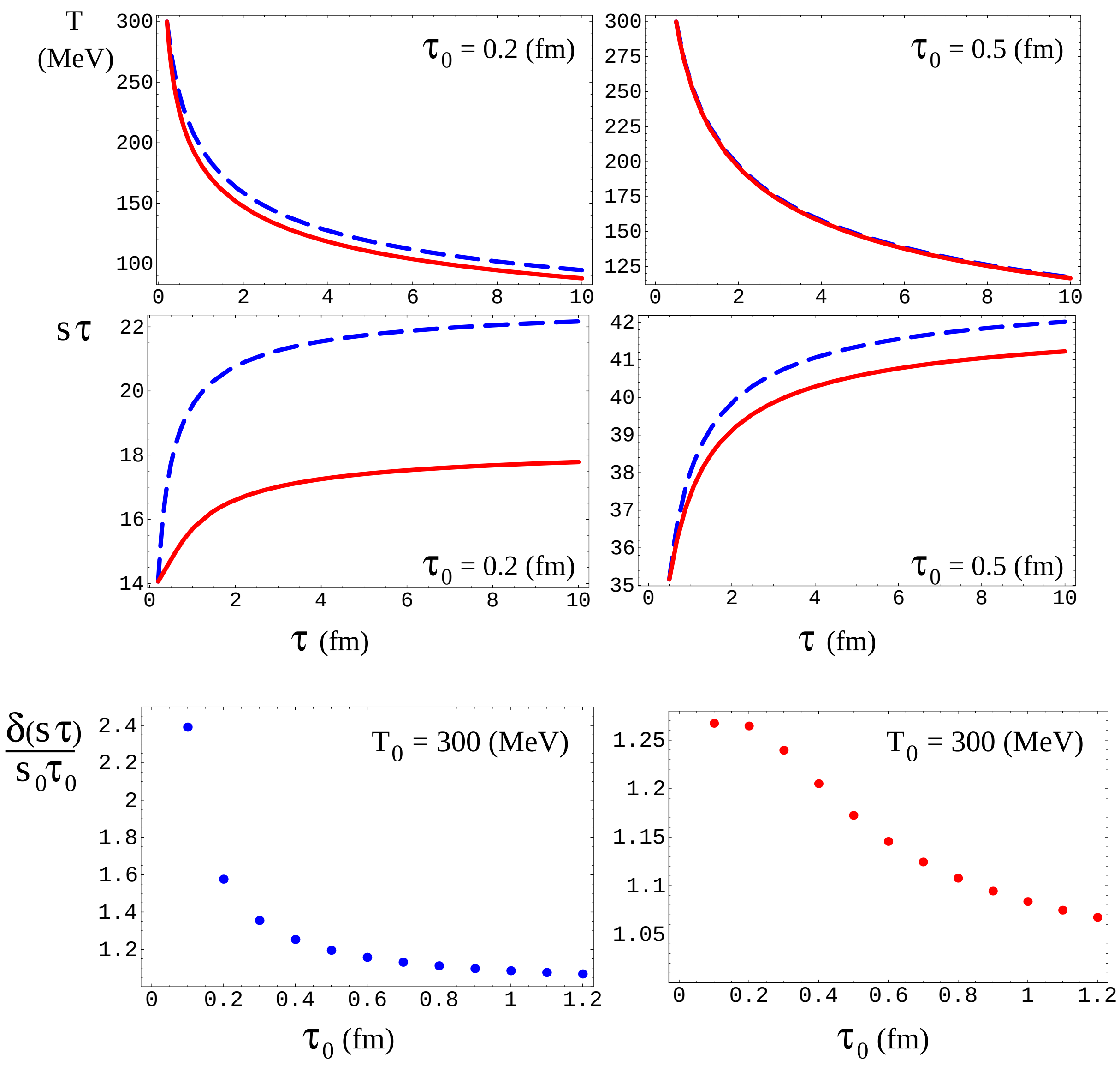
$$r_1 \longrightarrow \tau_R = 2 - \ln[2]$$

R. Baier, P. Romatschke, D.T. Son, A.O. Starinets, M.A. Stephanov, arXiv:0712.2451

‘ S. Bhattacharyya, V. E Hubeny, S. Minwalla, M. Rangamani, arXiv:0712.2456

$\beta_2 < 0$ while the IS second order hydro leads to $\beta_2 > 0$

How much entropy is produced by Hydro at RHIC?



Linearized Hydro to all orders

$$\Pi^{\mu\nu} = \Delta^{\mu m}\,\Delta^{\nu n}\; D_{mn,k}[\nabla]\, u^k$$

Tracelessness condition: $\quad \Delta^{mn}\; D_{mn,k}[\nabla]\, u^k = 0$

$$D_{mn,k}\, u^k = g_{mn}\left[\frac{2}{3}\eta_1 - \frac{1}{3}\eta_2\,\nabla^2\right](\nabla u) - \eta_1\left[g_{mk}\,\nabla_n + g_{nk}\,\nabla_m\right] u^k + \eta_2\,\nabla_m\,\nabla_n\,(\nabla u)$$

$$\eta_{1,2} = \eta_{1,2}[(u\nabla),\ \nabla^2] \;\rightarrow\; \eta_{1,2}[i\,\omega,\ \omega^2 - q^2] = \Re e\,\eta_{1,2} + \Im m\,\eta_{1,2}$$

$$\eta_1[\omega \rightarrow \mathbf{0},\ \mathbf{q} \rightarrow \mathbf{0}] \;\rightarrow\; \eta$$

We keep the nonlinear dispersion to all orders, but

We neglect nonlinear interactions (though some terms could be recovered).

Shear (Diffusive) channel:

$$G_R^T(\omega,\ q) \ = \ \frac{\eta_1\, q^2/2}{-\,i\,\omega \ + \ \eta_1\, q^2/2} \qquad\qquad \chi^R \ = \ \Im m\ G_R^T$$

Sound channel:

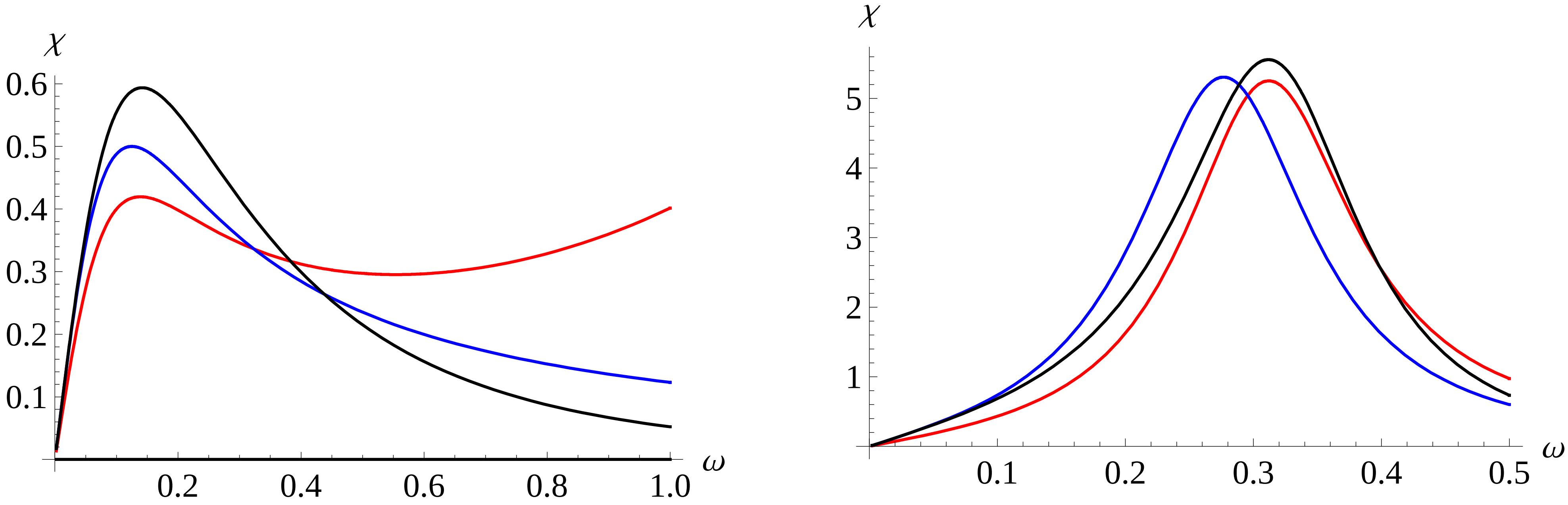
$$G_R^L(\omega,\ q) \ = \ \frac{2\,i\,\omega\,c^2\,q^2\,\tilde{\eta} \ - \ c^2\,q^2}{\omega^2 \ - \ c^2\,q^2 \ + \ 2\,i\,\omega\,c^2\,q^2\,\tilde{\eta}} \qquad\qquad \chi^L \ = \ \Im m\ G_R^L$$

$$\tilde{\eta} \ = \ \eta_1 \ + \ \eta_2\,(\omega^2 \ - \ 2\,q^2)/4$$

In order to extract $\eta_{1,2}$ we have to invert this relations.
For that we need both imaginary and real parts of the correlators.

Poles of the correlators should reproduce the entire tower of quasi-normal modes + their dispersion relations.

Shear $\quad\quad q = 0.5 \quad\quad$ **Sound**



red - AdS BH $\quad$ P. Kovtun, A. Starinets, Phys.Rev.Lett.96:131601,2006

blue - Navier Stokes $\quad$ black - IS second order hydro.

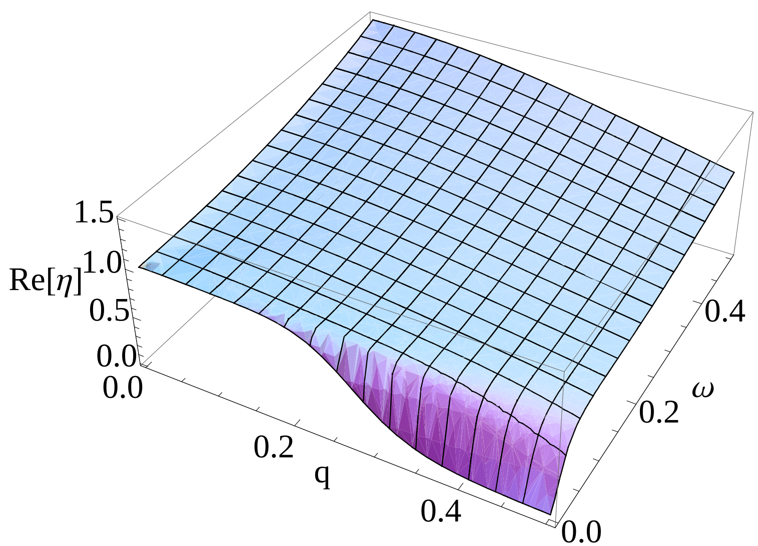
Discussion point:

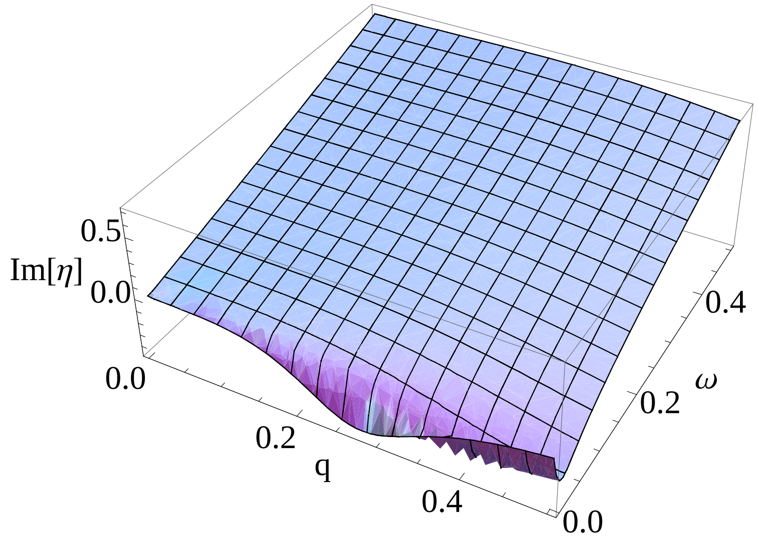
The spectral functions contain "non-thermal" vacuum physics, such as pair production.

Should this physics be removed when constructing hydro?

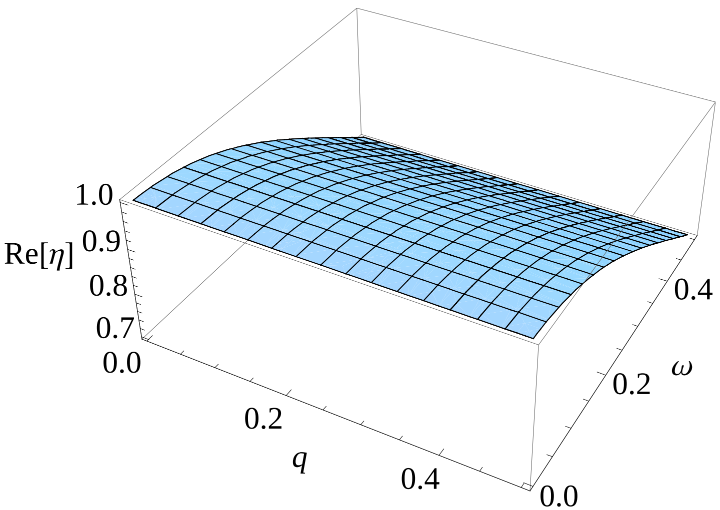
The "non-thermal" processes are real. They do occur in plasma.

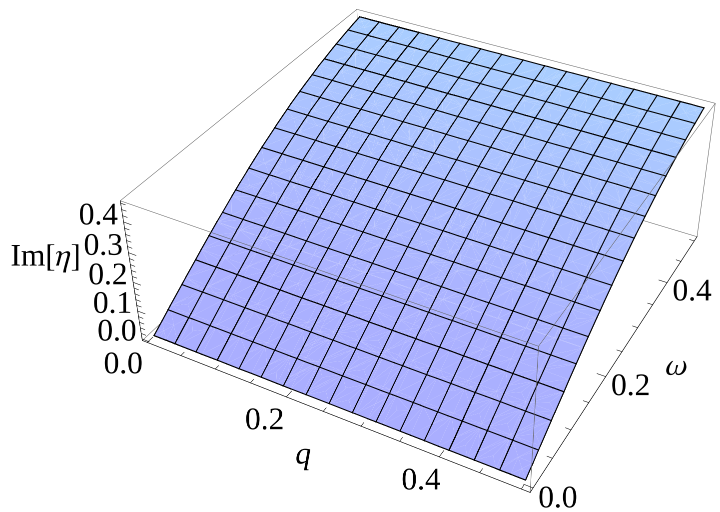
Should we model them as an effective hydro?





$$\eta_1 \; = \; 1 + i\,\tau_R\,\omega + \kappa\,q^2 + \lambda\,w^2\;\ldots \qquad \tau_R \; = \; 2 - \ln[2]\,, \qquad \kappa \; \simeq \; -1\,, \qquad \lambda \; \simeq \; 1.7$$





$$\bar{\eta} \; = \; 1 \; + \; i\,\tau_R\,\omega \; - \; \tau_R^2\,\omega^2\;\ldots$$

Concluding Remarks

- Higher order terms in the gradient expansion seem to be important at early times. Taking them into account is likely to reduce the dependence on the initial time of the evolution.

- IS second order hydro does not agree with the all-order hydro from the AdS/CFT. This hints that this second order hydro is potentially less trustable tool than it could be previously thought.